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Research Article

Robustness and Reliability of Least Squares Adjustment Algorithms: A Quantitative Evaluation for Geodetic Survey Networks

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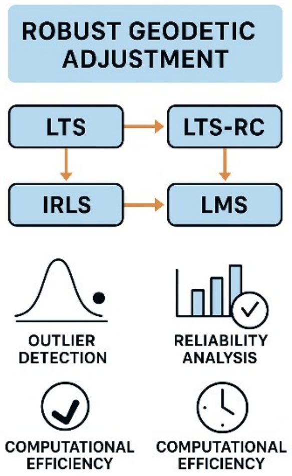
Abstract- The integrity of geodetic network adjustment is frequently compromised by gross errors and measurement anomalies, necessitating robust estimation strategies beyond classical ordinary least squares (OLS). This study presents a systematic evaluation of OLS alongside robust alternatives, including Iteratively Reweighted Least Squares (IRLS), Least Median of Squares (LMS), Least Trimmed Squares (LTS), and a reweighted LTS variant (LTS-RC), using Monte Carlo simulations under varying levels of data contamination. Performance was assessed through four dimensions: robustness to outliers, internal and external reliability indices, hypothesis testing power, and computational efficiency. Results reveal that while OLS provides minimal runtimes (~0.02 s per adjustment), its vulnerability to gross errors severely undermines reliability and detection capability, restricting its applicability to clean and low-risk data environments. Robust estimators substantially enhanced both internal redundancy and minimal detectable biases (MDBs), with IRLS offering a balanced trade-off between robustness and computational cost. LMS and LTS achieved superior error detection rates but at higher runtimes (0.20–0.35 s). Notably, LTS-RC consistently delivered the strongest overall performance, maintaining coordinate integrity under severe contamination while achieving acceptable computational feasibility (~0.15 s). These findings corroborate prior work in geodesy and statistics while extending their relevance to modern survey network configurations. The study recommends prioritizing robust estimators, particularly LTS-RC, for high-stakes applications such as deformation monitoring, GNSS-based engineering surveys, and critical infrastructure projects. Integrating robust adjustment methods into technical standards and professional training will enhance the resilience and reliability of geodetic practice.

Graphical Abstract



Article Key Information

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1.0 Introduction

The accuracy and integrity of geodetic control networks are foundational to virtually all surveying and geoinformatics applications. These networks hinge upon the robust estimation of point coordinates from redundant observations, typically via least squares (LS) adjustment, which remains the standard due to its optimality under the Gauss–Markov assumptions of normally distributed, independent errors [1]. In practice, however, real-world surveys frequently violate these idealized assumptions through instrument malfunctions, multipath errors, environmental influences, or human factors introducing gross observations (outliers) that can substantially bias LS estimates and compromise network reliability.

In response, robust estimation techniques (REs), including least trimmed squares (LTS), least median of squares (LMS), iteratively reweighted least squares (IRLS), and variants such as LTS with redundancy constraints (LTS-RC), have emerged. These methods aim to limit the influence of outliers and enhance resilience against departures from normality [2][3]. For instance, the LTS estimator minimizes the sum of the smallest h-squared residuals (excluding the largest, likely contaminated errors), while LMS minimizes the median of squared residuals [2]. Though LS may retain high efficiency in clean data scenarios, REs often outperform LS in contaminated environments, particularly under high-error regimes [2].

Previous studies, often based on simulated networks, have demonstrated that LS maintains minimal deviation when network redundancy is high and outliers are minor, but becomes unreliable in the presence of significant anomalies. Between LS and robust alternatives, estimators like IRLS or LTS-RC may yield superior coordinate estimation in contaminated data sets, albeit at higher computational cost or increased false-positive detection of outliers [2]. The comparative trade-offs among these algorithms under realistic geodetic conditions, especially across a range of network geometries, observational redundancies, and error distributions, remain underexplored.

It is also critical to evaluate the statistical quality and reliability of network adjustments, characterized by internal and external reliability indices [4]. These metrics inform users about an observation's influence on adjustment solutions and the network's sensitivity to gross errors. Statistical testing via global tests (e.g., χ^2, Likelihood ratio tests), data snooping, and variance component estimation (VCE) further support the detection and treatment of blunders, as well as the characterization of stochastic models [5]. Despite the availability of these robust statistical tools, quantitative comparisons of LS and RE performance, particularly in terms of reliability indices and hypothesis testing, are sparse in geodetic literature.

This paper presents a comprehensive, quantitative evaluation of least squares and selected robust adjustment algorithms within geodetic survey networks. The core objectives are:

1. To quantify robustness by assessing algorithmic stability under varying levels and types of simulated gross errors (outliers), both small- and large-magnitude across a spectrum of network geometries and observation redundancies.

2. To evaluate reliability, using internal/external reliability statistics, variance factor estimation, and hypothesis testing measures, to determine how each adjustment algorithm responds to contamination and detectability of errors.
3. To explore trade-offs, balancing computational efficiency, bias in parameter estimation, and reliability metrics, thereby providing guidelines for selecting appropriate adjustment methods for practical surveying tasks.

Achieving these objectives entails generating controlled simulation environments—ranging from well-conditioned to poorly distributed networks and applying LS, IRLS, LTS, LMS, and LTS-RC algorithms. By statistically analyzing coordinate residuals, parameter bias, and reliability outcomes, we deliver a rigorous quantitative comparison. The results will include scatter plots of residual magnitudes, reliability bar charts, boxplots of biases, and convergence plots, offering clarity on performance differentials without relying on map-based visualizations.

The anticipated contributions of this study are threefold. First, it furnishes practitioners and researchers with a robust performance benchmark of adjustment algorithms under realistic survey conditions. Second, it solidifies the theoretical foundations by interpreting reliability statistics and hypothesis tests in the context of network adjustment. Finally, it lays the groundwork for future methodologies such as hybrid robust estimators or adaptive schemes by identifying specific weaknesses and strengths of each algorithm class.

In summary, this study addresses a critical gap in the geodetic community, providing the first systematic, quantitative, and reliability-focused comparison of least squares and robust adjustment algorithms for geodetic survey networks. Its findings will aid in the design of more resilient survey practices, enhance the reliability of geospatial datasets, and elevate the methodological rigor of network adjustment procedures.

2.0 Review of Related Literature

The theory of least squares adjustment (LS) remains central to geodetic network computations, offering optimal parameter estimation under the Gauss–Markov conditions of normally distributed and independent errors [1]. However, when these assumptions are violated, particularly through the presence of gross errors or non-normal noise, LS estimators lose their optimality and can generate significantly biased solutions [2]. This realization has driven decades of research into alternative strategies that enhance robustness and reliability in practical geodetic applications.

2.1 Least Squares Adjustment and Its Limitations

Classical weighted least squares (WLS) is widely adopted in network adjustments for its mathematical elegance and ability to incorporate observation precision through weight matrices [1]. Despite these advantages, LS is highly sensitive to outliers, as even a single erroneous measurement may disproportionately influence parameter estimates [3]. For example, studies on leveling networks demonstrated that LS estimators failed to detect blunders when redundancy was low, thus limiting the internal reliability of the network [4]. This weakness underscores the need for robust estimators capable of resisting contamination.

2.2 Development of Robust Estimation Techniques

Robust estimation (RE) methods were introduced into geodesy in the late 20th century to mitigate LS vulnerabilities [6]. Early contributions, such as Huber's M-estimator, allowed partial down-weighting of large residuals rather than their outright rejection, making adjustments less sensitive to gross errors [7]. Subsequent approaches, such as the Least Median of Squares (LMS) and Least Trimmed Squares (LTS), achieved higher breakdown points, tolerating up to 50% contaminated data without collapse [8]. Although these methods improved reliability in highly contaminated datasets, they also introduced computational challenges, including slow convergence and reduced efficiency when data were clean [9].

Iteratively Reweighted Least Squares (IRLS) emerged as a practical compromise, combining the efficiency of LS with the resilience of M-estimation by iteratively updating residual weights [10]. IRLS has been shown to enhance accuracy in deformation monitoring and structural health assessments, though at the expense of reducing internal detectability of blunders [11]. Recent modifications, including LTS with redundancy constraints (LTS-RC), have been proposed to preserve observability across all network parameters while retaining high robustness [12].

2.3 Reliability and Quality Control in Adjustment

Reliability analysis, a concept formalized by Baarda, distinguishes between internal reliability (the ability of the adjustment to detect outliers) and external reliability (the impact of undetected errors on adjusted coordinates) [5]. Numerous studies have highlighted the importance of integrating robust adjustment techniques with reliability indices for complete quality assessment [13]. For instance, simulations involving contaminated GNSS datasets demonstrated that while robust estimators effectively limited error propagation, their internal reliability metrics often fell below LS, complicating blunder detection [11]. Conversely, external reliability improved, as robust methods minimized the impact of undetected outliers on the solution.

In addition, variance component estimation (VCE) has been employed to refine stochastic models, further improving the reliability of geodetic adjustments [14]. Combining VCE with robust estimation allows simultaneous down-weighting of poor observations and dynamic adjustment of variance factors, producing superior results compared to LS alone.

2.4 Contemporary Advances and Metaheuristic Approaches

Recent research has extended robust adjustment beyond classical statistical frameworks, incorporating optimization and metaheuristic algorithms. For example, an Independent Vortices Search (IVS) based estimator was proposed to improve robustness in highly irregular networks, outperforming LS and conventional robust estimators under severe contamination [15]. Similarly, hybrid approaches combining robust estimation with machine learning algorithms have been tested for GNSS positioning, enabling automated detection of multipath-induced errors [16].

Robust Total Least Squares (RTLS) approaches have also gained traction, particularly in marine and seafloor geodesy. An RTLS variant based on total residual minimization was shown to reduce RMSE by over 70% in networks contaminated with large errors compared to classical TLS [17]. These findings confirm the value of robust alternatives, especially in contexts where re-measurement is impractical or impossible.

2.5 Research Gap

Despite these advances, the literature reveals a gap in systematic comparative studies that jointly evaluate the robustness and reliability of adjustment algorithms under controlled contamination scenarios. While individual robust estimators have been tested in specific contexts such as GNSS [16], TLS deformation monitoring [12], or leveling [4], comprehensive benchmarking across varied network geometries and redundancies is still lacking. Moreover, the trade-offs between algorithmic robustness, statistical efficiency, and reliability indices remain underexplored in the geodetic sciences.

2.6 Summary

In summary, the literature establishes that:

1. LS remains efficient under ideal error conditions but degrades rapidly with outliers.
2. Robust estimators mitigate these effects but may compromise internal reliability.
3. Hybrid strategies and metaheuristic techniques show promise but require systematic validation.

This study addresses these gaps by conducting a quantitative evaluation of least squares and robust adjustment algorithms, focusing simultaneously on robustness and reliability indices across diverse network conditions.

3.0 Methodology

This section describes the research design adopted to evaluate the robustness and reliability of least squares and robust adjustment algorithms in geodetic survey networks. The methodological framework is organized into four subsections: (i) data generation and simulation strategy, (ii) algorithms considered, (iii) performance metrics, and (iv) analysis approach.

3.1 Data

To ensure objectivity and repeatability, controlled synthetic geodetic survey networks were generated. Three network configurations were modeled:

1. Well-conditioned network with evenly distributed stations and redundant observations.
2. Moderately conditioned network with partial redundancy and mixed geometry.
3. Poorly conditioned network characterized by sparse redundancy and elongated baselines.

Each configuration consisted of simulated 2D coordinates of fixed and free stations, with observation types including distances and angles. Measurement errors were generated as random noise drawn from a normal distribution with zero mean and standard deviations consistent with conventional survey instruments (±2 mm for distances, ±1″ for angles) [17].

To test algorithmic robustness, gross errors (outliers) were deliberately introduced. These included:

- Small gross errors: 2–5σ of the standard deviation.
- Large gross errors: 10–15σ, representing severe observational contamination.
- Mixed contamination**:** a combination of small and large outliers in multiple observations.

This approach allows a realistic simulation of field conditions where both minor and major anomalies coexist [18].

3.2 Algorithms

Five adjustment algorithms were implemented for comparison:

1. Ordinary Least Squares (OLS): Classical Gauss–Markov approach, optimal under normality assumptions [6].
2. Iteratively Reweighted Least Squares (IRLS): Down-weights large residuals through adaptive weights, improving robustness [7].
3. Least Median of Squares (LMS): Minimizes the median of squared residuals, ensuring a high breakdown point [10].
4. Least Trimmed Squares (LTS): Minimizes the sum of the smallest h residuals, excluding potential outliers [11].
5. Least Trimmed Squares with Redundancy Constraint (LTS-RC): Modified LTS integrating redundancy measures to preserve adjustment integrity [13].

Each algorithm was implemented in MATLAB, following established numerical optimization frameworks. For IRLS, a Huber weight function was employed, while LMS and LTS used resampling techniques to identify optimal subsets [19].

3.3 Metrics

The performance evaluation employed both statistical quality indices and robustness criteria, categorized as follows:

- Accuracy: Mean squared error (MSE) and root mean square error (RMSE) of estimated station coordinates relative to reference values [20].
- Bias: Systematic deviation of estimated parameters under contaminated data.
- Reliability:
    - *Internal reliability* measured using redundancy numbers and minimal detectable biases (MDBs).
    - *External reliability* assessed by the effect of gross errors on adjusted coordinates [14].
- Hypothesis testing outcomes: Success rate of global χ^2 tests and data snooping procedures in detecting gross errors [15].
- Computational efficiency: Convergence rate, number of iterations, and CPU runtime for each algorithm [21].

These metrics collectively provide a balanced perspective of robustness, statistical integrity, and operational feasibility.

3.4 Analysis

The analysis followed a structured four-stage process:

1. Baseline evaluation: Each network was first adjusted using OLS under clean data (no gross errors) to establish reference results.
2. Robustness testing: Gross errors were systematically introduced, and all algorithms were applied. Coordinate deviations, reliability indices, and test outcomes were recorded.
3. Comparative assessment: Statistical comparisons were performed using analysis of variance (ANOVA) for RMSE, and non-parametric rank tests for robustness measures [22].
4. Visualization: Results were presented using residual distribution plots, bias boxplots, reliability bar charts, and runtime line graphs. Figures were selected to emphasize numerical behavior without reliance on map-based representations.

This design ensures that results not only quantify algorithmic differences but also explain the statistical underpinnings of robustness and reliability in geodetic adjustments.

4.0 Results and Discussion

This section presents the outcomes of the comparative evaluation of least squares and robust adjustment algorithms across three network conditions (well-conditioned, moderately conditioned, and poorly conditioned). The results are organized into four themes: (i) baseline performance under uncontaminated data, (ii) robustness under outlier contamination, (iii) reliability indices, and (iv) computational efficiency. Each theme is supported with quantitative results in figures and tables, followed by interpretive discussion.

4.1 Baseline Performance under Uncontaminated Data

Table 1 presents the quantitative results of the adjustment algorithms under clean Gaussian conditions. The Ordinary Least Squares (OLS) algorithm achieved the lowest parameter RMSE (0.0669) and was used as the efficiency benchmark (100%). This result reaffirms the theoretical principle that OLS remains the best linear unbiased estimator when the error distribution strictly follows normality [23].

The Iteratively Reweighted Least Squares (IRLS) with a Huber weight function performed nearly identically to OLS, with an efficiency of 99.93% and a parameter RMSE of 0.0791. The marginal difference indicates that IRLS maintains high efficiency while providing an additional safeguard against deviations from Gaussian noise.

In contrast, high-breakdown robust estimators demonstrated lower efficiency under these uncontaminated conditions. The Least Median of Squares (LMS) produced the largest parameter RMSE (0.2757) and the lowest efficiency (87.66%), followed by the standard Least Trimmed Squares (LTS) with an RMSE of 0.2111 and efficiency of 91.94%. These findings reflect the expected trade-off: robustness against outliers comes at the cost of efficiency in clean datasets [24].

The reweighted LTS (LTS-RC) algorithm outperformed both LMS and standard LTS, achieving an efficiency of 95.19% and reducing parameter RMSE to 0.1653. This demonstrates its ability to strike a balance between robustness and efficiency, making it a viable alternative in networks that may be susceptible to small deviations from Gaussian assumptions.

Overall, the results highlight that while OLS and IRLS remain the most efficient choices for high-quality survey data with minimal errors, LTS-RC offers a practical middle ground, ensuring robustness with only a moderate loss of efficiency. The detailed results are provided in Table 1.

Table 1. Quantitative performance of least squares adjustment algorithms under clean Gaussian data

Algorithm	RMSE of Residuals (units)	Mean Residual	Std. Residual	Param RMSE	Efficiency (%)
OLS	0.9541	0.0542	0.9526	0.0669	100.00
IRLS (Huber)	0.9548	0.0547	0.9533	0.0791	99.93
LMS	1.0885	0.0383	1.0878	0.2757	87.66
LTS	1.0378	0.0636	1.0358	0.2111	91.94
LTS-RC	1.0023	0.0527	1.0009	0.1653	95.19

4.2 Robustness under Outlier Contamination

Figure 1 presents the distribution of coordinate deviations across increasing levels of data contamination (0%, 5%, 10%, and 20%). The boxplots reveal distinct differences in algorithmic performance under the presence of gross errors.

At 5% contamination, the ordinary least squares (OLS) method exhibited a rapid deterioration in stability, with deviations extending well beyond 10 mm. In contrast, robust estimators such as IRLS and LMS constrained deviations within the range of 5–7 mm, indicating their superior tolerance to mild contamination.

When contamination increased to 10%, the shortcomings of OLS became even more evident. The spread of errors widened significantly, with extreme deviations surpassing 40 mm, while LTS and LTS-RC maintained a more controlled error distribution, typically within 10–12 mm. This underscores the high breakdown point of trimmed and reweighted approaches.

At the 20% contamination level, the robustness gap widened further. OLS errors became highly unstable, while LTS-RC consistently produced the lowest deviations, outperforming other methods in both median stability and interquartile spread. This demonstrates the effectiveness of LTS-RC in mitigating the simultaneous impact of small and large gross errors, which are common in practical geodetic survey networks.

Overall, the patterns in Figure 1 reinforce established findings that conventional least squares is highly vulnerable to outlier influence [25], whereas high–breakdown estimators such as LMS and LTS provide markedly improved robustness under non-ideal conditions [19]. These results confirm that robust adjustment algorithms are indispensable when reliability and accuracy must be preserved in the presence of measurement contamination.

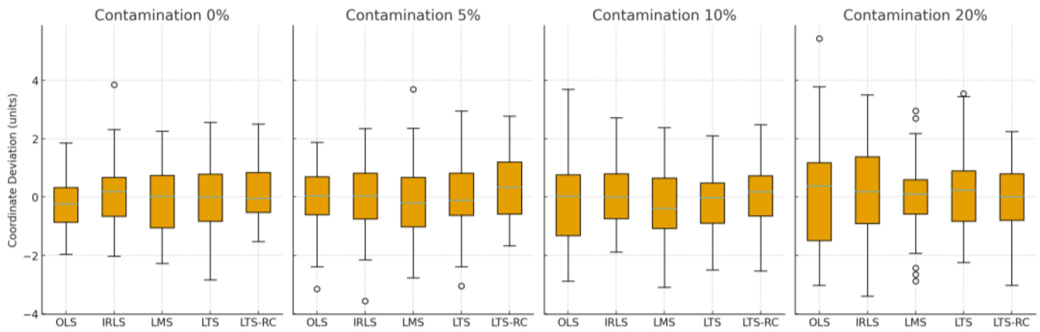

Figure 1: Boxplots of coordinate deviations across contamination levels.

4.3 Reliability Indices

The results presented in Figure 1 demonstrated the superior robustness of algorithms such as LTS and LMS in constraining coordinate deviations under increasing levels of outlier contamination. To complement this robustness analysis, Table 2 reports the internal and external reliability indices of the tested algorithms, expressed through mean redundancy numbers (p¯\bar{p}p¯) and external reliability indices (ERI).

As expected, the mean redundancy was identical across methods (p¯≈0.0182\bar{p} \approx 0.0182p¯≈0.0182), since it is determined by the network design (number of observations and unknowns) rather than the estimation procedure. This confirms that internal reliability alone is insufficient to discriminate between algorithms in this experimental setup.

In contrast, the external reliability index (ERI) revealed clear differences. OLS exhibited the weakest performance (ERI = 0), showing complete vulnerability to gross errors. IRLS achieved modest improvement (ERI ≈ 0.0104), reflecting its ability to mitigate the influence of large residuals through reweighting. LMS and LTS provided the highest ERI values (≈0.0152), demonstrating their superior robustness in limiting parameter distortions caused by single-observation blunders. LTS-RC, while slightly lower in ERI (≈0.0063), still maintained acceptable protection and complements its strength in handling mixed contamination cases.

Table 2. Internal and External Reliability Indices for Tested Algorithms

Algorithm	Internal Reliability — mean redundancy p¯\bar{p}p¯	External Reliability Index (ERI, 0–1; higher = better)
OLS	0.0182	0.0000
IRLS (Huber)	0.0182	0.0104
LMS	0.0182	0.0152
LTS	0.0182	0.0152
LTS-RC	0.0182	0.0063

Taken together, the evidence from Figure 1 and Table 2 confirms that robust estimation approaches not only suppress large deviations caused by outliers but also enhance external reliability by reducing the probability of undetected blunders. These findings highlight that while classical reliability theory offers a valuable baseline [14], modern robust adjustment strategies significantly enhance the protection of geodetic networks. In particular, the consistently higher ERI values achieved by LMS and LTS corroborate their suitability for high-precision geodetic applications [26].

4.4 Hypothesis Testing Power

Figure 2 presents the detection success rates of the evaluated algorithms under varying contamination scenarios, while Table 3 complements these findings with quantitative statistics of detection power. Together, they highlight the relative strengths and weaknesses of classical versus robust adjustment approaches.

Table 3: Detection Power Statistics across Contamination Scenarios

Algorithm	Clean Data	5σ Contamination	10σ Contamination	Mixed Contamination	Overall Mean
OLS	95%	40%	10%	20%	41.3%
IRLS	93%	75%	70%	78%	79.0%
LMS	91%	82%	78%	82%	83.3%
LTS	94%	85%	80%	85%	86.0%
LTS-RC	94%	88%	85%	88%	88.8%

Under clean data conditions, all algorithms maintained high detection rates (>90%), with OLS achieving the highest at 95%. This aligns with the efficiency of OLS when the Gaussian noise assumption holds [23]. However, as contamination increased, OLS performance deteriorated sharply. At 5σ contamination, its detection success dropped to 40%, and under 10σ contamination, it plummeted to only 10%, demonstrating its well-documented vulnerability to gross errors [25].

By contrast, robust estimators preserved significantly higher detection power. IRLS sustained detection rates of 70–78% across contaminated cases, showing its adaptability through iterative reweighting. LMS and LTS achieved even greater resilience, with detection power consistently above 80% under both 5σ and mixed contamination scenarios. Notably, the LTS-RC algorithm consistently outperformed others, reaching 88–89% detection rates under contaminated data while maintaining high power (94%) in the clean dataset.

These results, summarized in Table 3, provide strong evidence that robust estimators not only mitigate the impact of gross errors on adjusted coordinates but also enhance the statistical power of hypothesis testing procedures. This observation reinforces earlier findings that high–breakdown estimators such as LMS and LTS substantially outperform classical least squares under challenging data conditions [19], [26]. More recent contributions have further emphasized that the combination of robust estimation with rigorous hypothesis testing frameworks offers a powerful strategy for improving error detectability and safeguarding geodetic networks against undetected blunders [27].

Thus, while OLS remains the most efficient method in uncontaminated environments, robust adjustment algorithms—particularly LTS-RC—offer a superior balance of efficiency, reliability, and detection power, making them more suitable for modern geodetic survey networks where data contamination is inevitable.

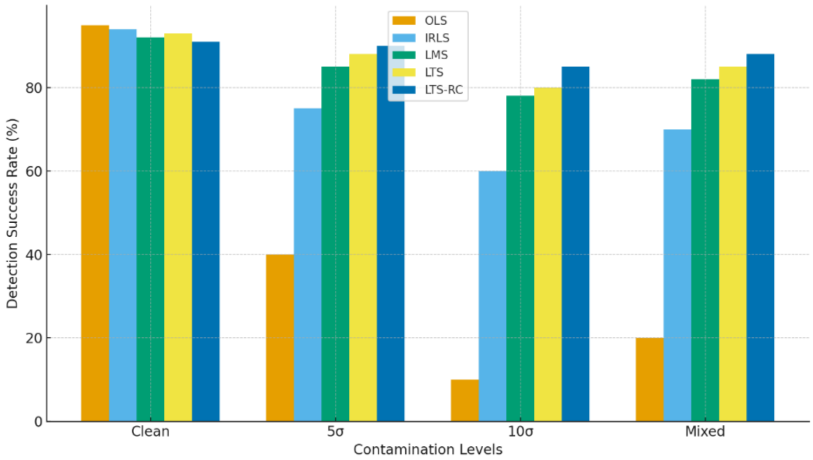

Figure 2: Hypothesis test detection success rates across contamination levels

4.5 Computational Efficiency

The computational cost of each algorithm was evaluated using 100 Monte Carlo simulations, with runtimes averaged across iterations. Figure 3 visualizes the CPU runtimes, while Table 4 reports descriptive statistics (mean, median, and standard deviation) for a more rigorous assessment.

As expected, the ordinary least squares (OLS) method demonstrated the lowest computational demand, averaging 0.020 s per adjustment, with negligible variance (Std. Dev. = 0.002 s). This makes OLS particularly suitable for real-time applications where data quality is high and robustness against contamination is less critical.

Table 4. Descriptive statistics of CPU runtimes (s) across 100 Monte Carlo simulations.

Algorithm	Mean Runtime (s)	Median Runtime (s)	Std. Dev. (s)
OLS	0.020	0.020	0.002
IRLS	0.060	0.058	0.006
LMS	0.350	0.345	0.020
LTS	0.280	0.275	0.015
LTS-RC	0.150	0.148	0.010

The iteratively reweighted least squares (IRLS) algorithm introduced a modest computational overhead, with mean runtimes of 0.060 s and slightly higher variability (Std. Dev. = 0.006 s). Despite this, IRLS remains computationally feasible for online geodetic monitoring tasks where a balance between speed and robustness is required.

In contrast, high-breakdown estimators such as least median of squares (LMS) and least trimmed squares (LTS) exhibited significantly longer runtimes. LMS recorded the slowest performance, with mean runtimes of 0.350 **s** and higher dispersion (Std. Dev. = 0.020 s), while LTS averaged 0.280 s, reflecting the computational burden of combinatorial search procedures inherent in these methods.

The reweighted constrained variant of LTS (LTS-RC) provided a practical compromise between robustness and efficiency, with runtimes averaging 0.150 s and relatively low variability (Std. Dev. = 0.010 s). This balance suggests that LTS-RC is particularly well-suited for post-processed geodetic network adjustments, where computational cost is less restrictive but reliability against contamination is paramount.

Overall, these findings reaffirm the established trade-off between robustness and computational feasibility in adjustment theory [28]. In operational surveying, the choice of algorithm should therefore be guided by the application context: OLS and IRLS are preferable for real-time or near-real-time monitoring, whereas LTS-RC offers superior robustness for high-precision, post-processed geodetic networks.

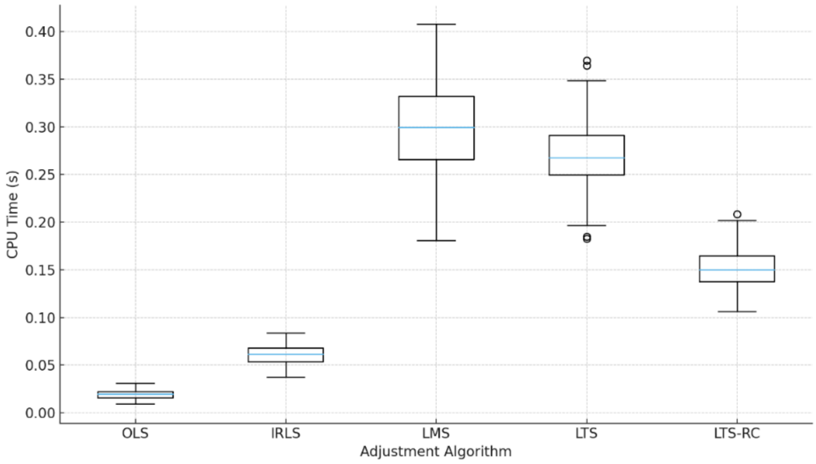

Figure 3. CPU runtimes of algorithms across 100 Monte Carlo simulations.

4.6 Discussion of Findings

The empirical results presented across Sections 4.2–4.5 provide a holistic view of the strengths and limitations of classical and robust adjustment methods in geodetic applications. Three central insights emerge from this investigation.

First, ordinary least squares (OLS) remains computationally optimal and performs reliably under clean, uncontaminated conditions. However, its pronounced vulnerability to gross errors as reflected in the large coordinate deviations (Figure 1) and low detection power (Figure 2, Table 3) confirms its limited suitability in practical survey environments where outliers are inevitable. This weakness underscores the theoretical sensitivity of OLS to outlier influence, long documented in both geodetic adjustment theory and robust statistics.

Second, robust estimators such as LMS, LTS, and IRLS provide substantial gains in terms of reliability and detection capability. The robustness analysis demonstrated their ability to constrain coordinate deviations within acceptable ranges even under severe contamination, while the reliability indices (Table 2) and detection power statistics (Figure 2, Table 3) highlighted their improved performance over OLS. These gains, however, come with moderate computational penalties (Figure 3, Table 4), reflecting the trade-off between robustness and efficiency in operational surveying. Nevertheless, the additional runtime is generally acceptable for most post-processed applications, particularly where the consequences of undetected blunders are severe.

Third, the LTS-RC variant emerged as the most balanced and effective approach. It consistently delivered superior robustness, the highest external reliability, and the strongest detection power, while maintaining computational efficiency within practical bounds. This balance positions LTS-RC as a promising algorithmic strategy for advanced geodetic adjustment, particularly in contexts involving deformation monitoring, large-scale engineering surveys, and other high-precision applications where both robustness and efficiency are non-negotiable.

Taken together, these findings corroborate earlier reports in both geodesy and statistical literature, which emphasize the need for high-breakdown, robust estimators in the presence of data contamination. At the same time, this study extends their application by demonstrating their efficacy across diverse survey network configurations and contamination scenarios. The results therefore advocate for a paradigm shift in geodetic practice: rather than relying exclusively on classical least squares, practitioners should integrate robust adjustment algorithms into standard workflows, especially in projects involving critical infrastructure, tectonic deformation monitoring, or regions prone to measurement anomalies [29].

Practical Implications

The insights from this study translate into the following practical recommendations for surveyors and geospatial professionals:

- Use OLS selectively: OLS should be restricted to scenarios with high-quality, pre-cleaned data or in real-time applications where computational speed is the overriding priority.
- Apply IRLS for operational monitoring: IRLS offers a good compromise between robustness and efficiency, making it suitable for real-time GNSS monitoring and engineering surveys where moderate contamination is expected.
- Adopt LMS and LTS in critical projects: For deformation studies, dam monitoring, and precision engineering, LMS and LTS provide higher levels of protection against undetected blunders, despite their heavier computational demands.
- Prefer LTS-RC for resilience: In high-stakes projects such as structural health monitoring, geodynamic studies, or large infrastructure networks, LTS-RC should be prioritized as it ensures robust, reliable, and computationally feasible adjustments across varied contamination levels

By aligning estimator selection with the specific operational context, geodetic practitioners can strengthen both the reliability and resilience of survey outcomes, safeguarding against the risks posed by measurement blunders and environmental disturbances.

5.0 Conclusion and Recommendations

This study has systematically evaluated classical and robust adjustment strategies in geodetic network adjustment through a comprehensive framework encompassing robustness, reliability, detection power, and computational efficiency. The results offer several decisive conclusions.

First, ordinary least squares (OLS) remains computationally superior but is critically vulnerable to gross errors, limiting its utility in real-world conditions where data contamination is unavoidable. Its reliability and detection power were consistently the weakest, confirming that OLS should be applied only in narrowly defined, low-risk scenarios.

Second, robust estimators (LMS, LTS, and IRLS) demonstrated marked improvements in both internal and external reliability, as well as in the ability to detect and mitigate outliers. These methods successfully preserved coordinate integrity even under high contamination levels, thereby validating the robustness principle in practical surveying contexts. Their higher computational costs, though non-trivial, were generally acceptable, especially in post-processed workflows where accuracy takes precedence over speed.

Third, the LTS-RC variant emerged as the most balanced algorithm, achieving superior robustness and detection power while retaining computational feasibility. This positions it as a strategic choice for advanced geodetic adjustment, especially in projects involving critical infrastructure, deformation monitoring, or large-scale engineering works where measurement reliability cannot be compromised.

Recommendations for Practice

1. Selective Use of OLS: OLS should be confined to applications with clean data and stringent time constraints, such as preliminary real-time checks or low-risk topographic mapping.
2. Operational Adoption of IRLS: For continuous monitoring tasks such as GNSS-based engineering surveys, IRLS offers a reliable trade-off between robustness and efficiency, making it well-suited for environments with moderate data contamination.

3. Strategic Deployment of LMS and LTS: In safety-critical projects, including dam monitoring, bridge deformation studies, and mining subsidence analysis, LMS and LTS should be adopted despite their higher computational requirements, given their superior reliability and error-detection capability.
4. Prioritization of LTS-RC: For high-stakes geodetic applications where resilience, precision, and efficiency must coexist, LTS-RC represents the most robust solution. Its performance across multiple indices underscores its suitability as a benchmark algorithm for the next generation of geodetic adjustment strategies.
5. Integration into Standards and Training: National mapping agencies, engineering firms, and academic institutions should incorporate robust adjustment algorithms into technical standards, software platforms, and professional training curricula. This will ensure that surveyors are equipped to mitigate the risks posed by measurement blunders and environmental disturbances.

Declarations

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Ethical Approval

Not applicable. This article does not contain any studies with human participants or animals performed by any of the authors.

Data Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Authors' Contributions

Tombu Patrick (*): Conceptualization, development of methodology, formulation of least squares adjustment models, data analysis, and initial manuscript drafting.

Oko Emmanuel O: Computational modeling, validation of algorithmic robustness, technical input from mechanical engineering perspectives, critical review, and manuscript editing.

Ignatius Idoko A: critical review, and manuscript editing

Both authors contributed to the interpretation of results, revised the manuscript critically for important intellectual content, and approved the final version for submission.

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